Notch Effects, Stress State, and Ductility

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A review of the literature on testing of notched specimens is provided with emphasis on short-term notched bar tension tests. The effects of notch geometry, stress state, and smooth-bar tensile properties on notched ductility, notch sensitivity, and mode of fracture are discussed. For design against failure due to notch weakening, a simple criterion based on a limiting value of notched ductility is proposed. Notched ductility, for the metals considered in this study, can be approximated by the reduction in area from a smooth tension test divided by a tensile stress triaxiality factor that is proportional to the ratio of hydrostatic to octahedral shearing stresses associated with the notch.

Introduction

The introduction of a notch in a test specimen or design component results in stress concentrations, a state of triaxial stresses, and reduced ductility. Notched-bar impact, tension, and bending tests have been performed on metals for over a century in attempts to understand the response of metals to notches and stress concentrations. The results have been used to describe a number of design parameters including impact toughness, notched ductility, fracture susceptibility, notch sensitivity, fatigue strength with stress concentrations, and notch creep-rupture.

Notched specimen impact testing was performed as early as the mid-nineteenth century [1]. In these early tests, the brittleness of a specimen was recognized by lack of significant deformation and by the "crystalline"[1] appearance of the surface of fracture [4]. Russel [5] in 1897 tested notched specimens in three point bending and measured energy absorption by means of an impact testing machine. Controlled impact testing on notched rectangular bars was developed at the beginning of the twentieth century much through the work of Izod [6] in 1903, Charpy [7] in 1909, and others.

The use of cylindrical notched specimens in impact testing was proposed by Philpot [8] in 1918. Although the cylindrical geometry did not replace rectangular bars in impact testing, it did become popular for use in notched tension testing. The effect of notches on the tensile behavior of cylindrical steel specimens was described by Ludwik and Scheu [9] in 1923. Emphasis was placed on the role of triaxial tensile stresses in restricting plastic flow and thus promoting the ductile-to-brittle transition in steels. Cylindrical specimens of mild steel of varying notch depth and notch radii were tested under tensile loading. The increase in tensile strength and decrease in notched ductility observed were attributed to the presence of transverse stresses (and thus triaxiality), the magnitude of which were assumed to increase with the sharpness of the notch.

During the 1920's and 1930's, much debate centered on the role of other variables on the ductile-brittle transition in notched specimens. Based on both notch impact tests and notched tension tests, the effects of geometry [10–15], strain rate [12, 15, 16, 23, 24], temperature [14, 17–23, 25], heat treatment [14, 17, 20, 22], and composition [14, 19–23] were investigated. Although the effects of notches on the fatigue failures of railway axles were recognized by Rankine as early as 1843 [2], considerable fatigue testing of notched specimens was not performed until the early twentieth century [25–28].

During the first half of the twentieth century considerable advances were made in obtaining elastically calculated stress distributions around points of stress concentrations and notches [29]. Heyn [30] in 1921 describes optical experiments based on the investigations of Kirsch, Inglis, and Coker to determine the effects of holes and notches on the distribution of stress. Of particular interest were the elastic stress distributions obtained by Neuber [31] in 1937 for bars containing notches of simple idealized shapes. His early solutions applied only to elliptical internal notches and hyperbolic external notches under tensile loading conditions. Neuber's later publications, however, included solutions for more complicated conditions, such as the bending and torsion of notched bars of rectangular cross-section.

During the 1940's much attention was placed on the effects of these elastically calculated stress concentrations or "stress raisers" on engineering design. Stress raisers [32] were attributed to a number of notch geometries, such as surface dents, grooves, holes, keyways, and section changes. It was also recognized that inherent external and internal notches can be caused by slag inclusions, metal defects, and graphite flakes in cast iron. With the increasing use of welded construction during this period, the notch sensitivity due to welds and welding defects was also of concern [33]. Much of the notched tension testing performed

[1]Rankine [2] in 1843 and Kirkaldy [3] in 1861 refuted the idea that metals failed because they had crystallized. Even after Kirkaldy's published arguments that a fracture surface could have either a fibrous or crystalline appearance solely by altering the shape of the specimen or varying the strain rate, the misconception that the metal actually "crystallizes" persists even in the present.

Contributed by the Materials Division for publication in the JOURNAL OF ENGINEERING MATERIALS AND TECHNOLOGY. Manuscript received by the Materials Division June 12, 1978; revised manuscript received July 7, 1978.

around World War II was done by Sachs at the Case School of Applied Science and by Lubahn at the General Electric Research Laboratory [32, 34–42].

The advent of higher temperature machinery after World War I for more efficient energy conversion and for improved chemical production capabilities led to the development of high temperature alloys and growing concern about temperature effects on engineering design [43]. The phenomenon of embrittlement of steel subject to high temperatures was recognized along with the potential of notches and high tensile stresses in accelerating this embrittlement. In 1952, much of the information on notch effects at elevated temperatures was collected at the fifty-fifth annual meeting of the American Society for Testing Materials. Based on notched tension tests at elevated temperatures over an extended time period, the influence of notches on the static and rupture strength of high temperature metals was addressed [44–52]. The recognized influence of numerous variables such as geometry, temperature, strain rate, metallurgical state, notch preparation, composition, time, mode of fracture, and ductility, along with the contradictory results of some of the investigations, emphasized the extreme complexity of the subject. Although still in a limited state of knowledge, loss of ductility at elevated temperatures, compounded by the embrittling effects of notches, continues to be of concern in such applications as nuclear reactors and high temperature turbines.

Due to introduction of cryogenic propellants such as liquid oxygen and liquid hydrogen (boiling points −183°C and −253°C, respectively) in missiles and space vehicles, the properties of high-strength structural materials at extremely low temperatures became of extreme importance to the aerospace industries in the 1950's. At reduced temperatures, some metals ordinarily classified as ductile become brittle, and the effects of stress concentrations due to welding defects, tool marks, assembly eccentricities, sharp corners, and notches can be significant. The number of low-temperature notched tension tests performed during the 1950's and 1960's attests to the growing concern about notch effects in these applications [53–71]. Another interesting application of notched tension tests at low temperature involved feasibility studies of nuclear powered rockets. The combined effects of low temperature and irradiation caused even ductile 300-series austenitic stainless steels to be notch sensitive (i.e., notched tensile strength lower than unnotched tensile strength) [72].

The involvement of the areospace industry in notched tension testing has not been limited to cryogenic applications. In order to maximize strength-to-weight ratios, relatively light, high strength metals are often subject to severe tensile loading conditions with potential for large stress concentrations. Notched tension tests on titanium, aluminum, and magnesium alloys have been used as screening tests in numerous alloy development programs. The application of much of this work has been to correlate plane-strain fracture toughness with notched tensile properties [73].

Most of the applications of notched tension tests have been in attempts to design against brittle fracture. Notched tension testing decreased in popularity after Irwin's 1948 publication on fracture dynamics [74] with the associated increasing interest in energy approaches to fracture. Recent investigators, however, have used notched tension tests to study the influence of stress state on ductile fracture. McClintock [75] and Rice and Tracey [76] have shown that void growth associated with ductile failure depends strongly on stress state. Tensile test specimens of notched cylindrical geometry provide an attractive means of introducing triaxial tension and of measuring failure parameters. The goal of current work in this area is to understand better ductile failure initiation and to develop criteria to prevent it [77].

Notch Geometry and Stress State

Notched-bar tension tests are performed on notched specimens of either rectangular cross-section, with varying thicknesses, or circular cross-section, with varying radii. For either type of specimen, the stress state will vary with the notch depth and notch root radius. The notch contour is classified as shallow or deep, external or internal, single or multiple, and blunt or sharp.

The most comprehensive collection of elastically calculated stress concentrations and stress distributions around notches is provided in the work of Neuber [31]. For all geometries studied, at the notch root the transverse stress is zero and the longitudinal or axial stress is a maximum. As shown in Fig. 1 for the external deep circumferential notch under tension, the tangential stress "σ_2" and the radial stress "σ_3" are approximately equal at points away from the proximity of the notch root.

Bridgman in 1943 [78] obtained a plastic stress distribution of the neck of a tension specimen by applying the von Mises conditions of plasticity. Because he approximated the contour of a tension specimen and the lines of principal stress at the neck by a circle, the Bridgman solution has been used to approximate the stress distribution in tensile specimens with circular notches (see Fig. 2). Recent numerical solutions obtained by Benzley, et al. [77] indicate that the Bridgman solution approximates the stress state of bluntly notched cylindrical specimens, loaded beyond general yielding, relatively well. The Bridgman solution, however, appears to provide a poor approximation of the stress state of sharply notched specimens because of the extreme strain gradients at the notch tip. It appears that finite element techniques must be used to correctly model the nonlinear material behavior and the large but finite strains associated with sharp notches.

Notch Sensitivity

The tendency for reduced ductility in the presence of a triaxial stress field and steep stress gradients is often termed "notch sensitivity" [79], and a material is classified as notch sensitive or notch weakened if its notch strength ratio (NSR) is

Nomenclature

d = net diameter in notched specimen
NSR = notch strength ratio, defined in equation (1)
r = notch root radius
RA = reduction in area in a smooth specimen tension test
(RA_n) = notched reduction in area
S_u = ultimate strength
$(S_u)_n$ = notched ultimate strength
TF = triaxiality factor, defined in equation (2)
σ = stress
$\sigma_1, \sigma_2, \sigma_3$ = principal stresses
σ_m = mean stress = $\dfrac{\sigma_1 + \sigma_2 + \sigma_3}{3}$
$\bar{\sigma}$ = effective stress

$$= \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2}$$

In notched specimens,

σ_1 = axial or longitudinal principal stress
σ_2 = tangential principal stress
σ_3 = radial principal stress

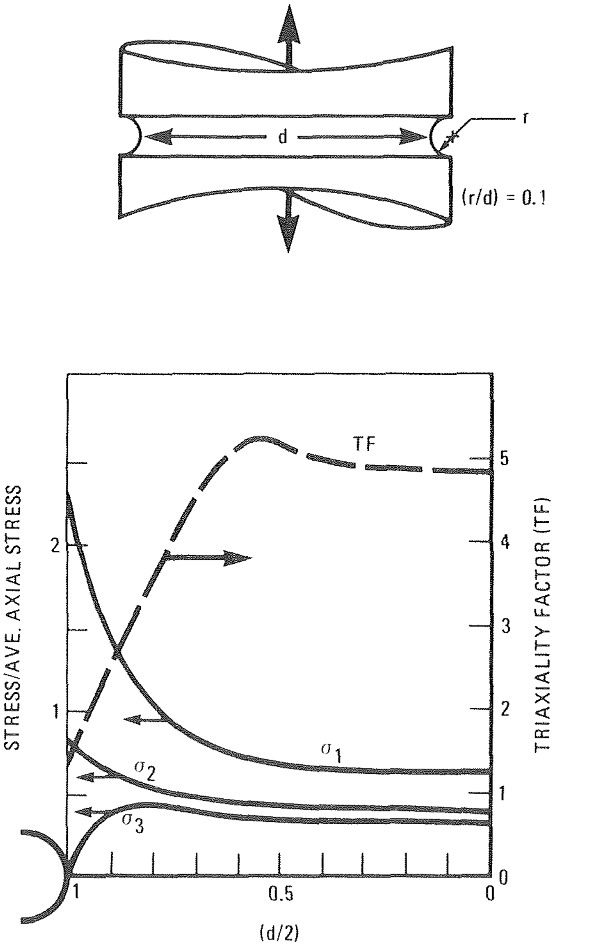

Fig. 1 Variation of elastically-calculated stress state across a notched round bar in tension (courtesy of D. V. Nelson, GE-ARSD)

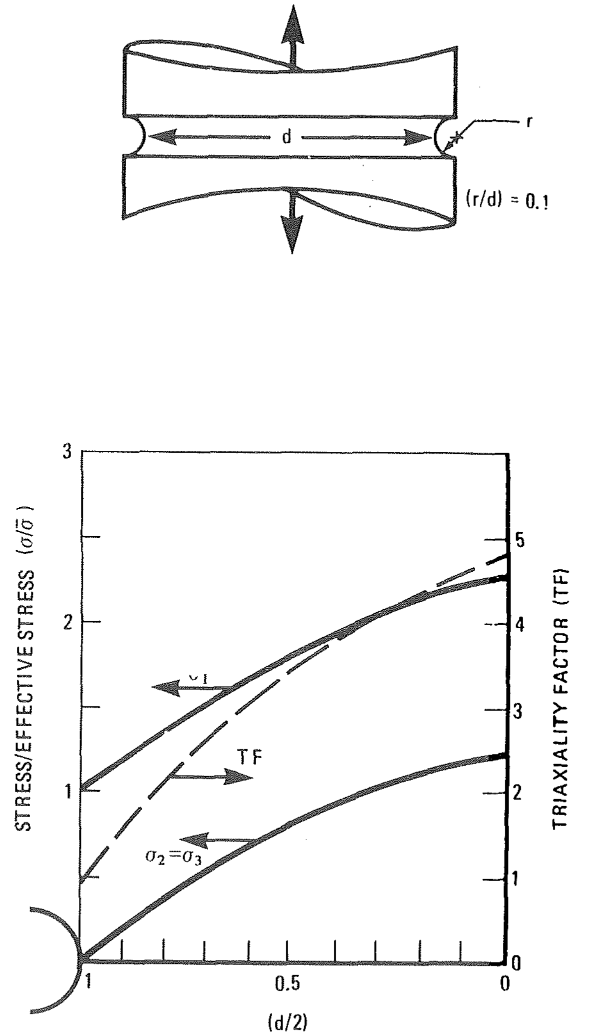

Fig. 2 Bridgman stress distribution across a notched round bar in tension (courtesy of D. V. Nelson, GE-ARSD)

less than unity:

$$\mathrm{NSR} = \frac{(S_{un})}{S_u} \qquad (1)$$

where: NSR = notch strength ratio $\begin{cases} >1 \text{ notch strengthened} \\ <1 \text{ notch weakened} \end{cases}$

(S_{un}) = ultimate tensile strength in notched specimen

S_u = ultimate strength in smooth tension specimen.

McClintock [80] in 1961 proposed that the maximum load calculated from a plastic analysis for nonstrain hardening materials be used as a standard by which actual behavior of notched specimens could be compared. McClintock reports that, theoretically, for a deep enough notch, the notch strength ratio for a cylindrical specimen might be as large as 2.7. In other words, the load carrying capacity of a notched cylindrical specimen could be 2.7 times that of a corresponding smooth specimen of the same cross-sectional area if enough ductility were present in the material to adequately accommodate plastic flow. However, as reported by McClintock and Irwin [81], the maximum notch strength ratio reported in the literature of notched tension testing is around 1.7. For example, consider the recent experimental work on ductile failure initiation by MacKenzie, et al. [82]. The maximum notch strength ratio achieved from these tests is around 1.7 for a high strength steel with the chemical composition of HY-130 and a smooth bar reduction in area of 74 percent.

The work done by Fried and Sachs [38] on annealed AISI 1025 silicon killed steel (smooth-bar reduction in area of 55 percent) shows that the notch strength ratio of notched cylindrical specimens will increase with notch sharpness up to a certain point ($\simeq$ 1.3—1.5) and then decrease for greater notch sharpness. Associated with the shift from increasing to decreasing notch strength ratio with increasing notch sharpness is a shift in the location of fracture initiation from the center for bluntly notched specimens to the notch tip for the sharply notched specimens.

Christian [71] shows that ductile 300-series stainless steels (301, 304, and 310) and 2014-T6 aluminum at room temperature can be made notch sensitive by using extremely sharp notched specimens with an elastic stress concentration factor of 19. Thus it must be concluded that notch sensitivity is not a material property but will vary with both material properties (such as ductility and strain hardening capacity) and geometry. As will be discussed in the next section of this paper, the geometrical considerations and size effects of notched tension specimens may be explained, at least in part, by considering the notched state of stress, which will vary with notch sharpness and depth.

Ductility and Stress State

It has long been recognized that stress state can affect ductility. As shown by the early work of Ludwik and Scheu [9],

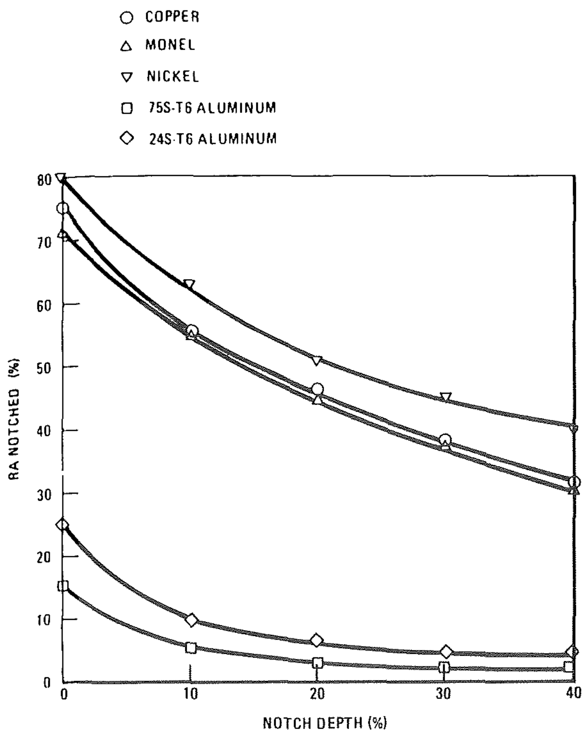

Fig. 3 Notched ductility as a function of notch depth for a variety of face-centered cubic metals in liquid Nitrogen (−196°C)

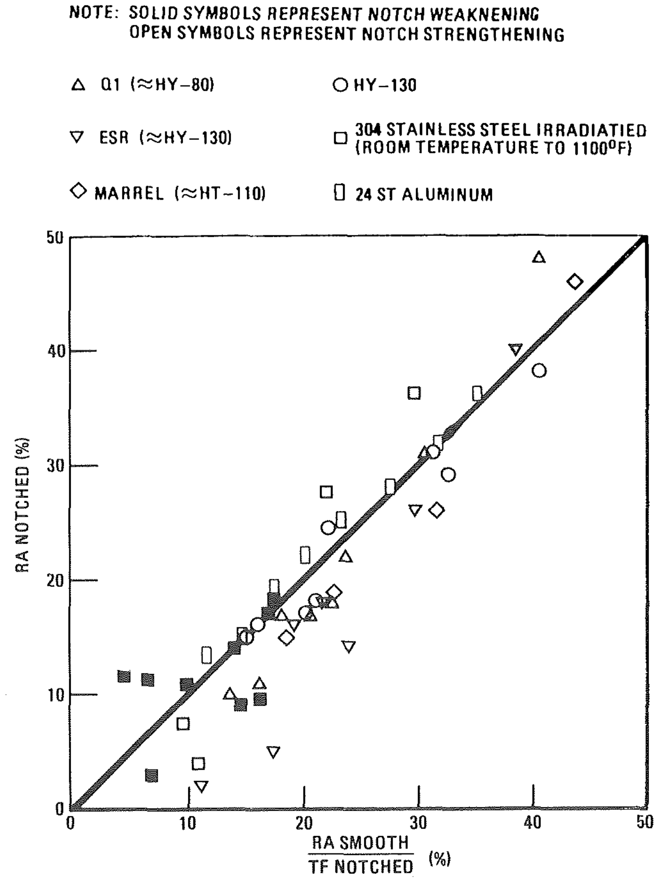

Fig. 4 Reduction in area of notched specimens versus reduction in area of corresponding smooth tension specimens divided by triaxiality factor for notched specimens

tensile stress triaxiality tends to reduce fracture ductility. The opposite trend is observed for compressive stresses. The work by Bridgman [78] shows that the ductility of a material, tested under combined tension and high hydrostatic pressure, is increased by increasing the pressure.

Traditionally the degree of stress triaxiality has been defined as the ratio of σ_3/σ_1 where σ_1 and σ_2 are the maximum and minimum principal stresses, respectively. This has the disadvantage that not all three principal stresses are represented. The following triaxiality factor introduced by Davis and Connelly [83] will be used to describe the state of stress for the purposes of this paper:

$$\mathrm{TF} = \frac{\sqrt{2}(\sigma_1 + \sigma_2 + \sigma_3)}{[(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2]^{1/2}} = \frac{3\sigma_m}{\bar{\sigma}} \quad (2)$$

where: σ_1, σ_2, σ_3 = principal stresses.

This triaxiality factor can be considered as a measure of the ratio of hydrostatic to octahedral shearing stresses. The triaxiality factor will become +1 for uniaxial tension, 0 for pure shear, and −1 for uniaxial compression. Fig. 1 shows the elastically calculated axial, radial, and circumferential stresses along with the associated triaxiality factor for a notched round bar in tension.

To illustrate how triaxial stress states develop in notched specimens, consider the notched cylindrical bar in Fig. 1. Under tensile loading, the small volume of highly stressed material near the notch tends to deform plastically at a lower load than the material in the regions of larger diameter. The bulk of the less stressed material away from the notch restricts plastic flow, that is, it prevents the radial distortion which accompanies the axial strain ("Poisson effect"), and a state of tensile triaxial stress is developed. This restriction to plastic flow contributes to the reduced ductility measured in notched tension specimens at fracture.

Over the years, many investigators have considered the effects of stress state on fracture ductility [e.g., 9, 37, 77, 78, 82–89]. Ripling [84] tested six face-centered cubic metals and alloys (copper, AISI 310 stainless steel, monel, nickel, and 24S-T4 and 75S-T6 aluminum) and measured notched ductility (notched reduction in area at fracture) for sharply notched specimens (notch radius < 0.03 mm) with varying notch depths. The test results illustrated in Fig. 3 show decreasing notched ductility with increasing notch depth and thus increasing tensile stress triaxiality. Manjoine [87] has proposed that elevated temperature failure strain should vary inversely with the triaxiality factor defined in equation (2). Recently the use of the triaxiality factor to account for the effect of stress state on fracture ductility has been utilized by Nelson, et al. [85] in a strain limit intended to protect against localized cracking in low ductility material.

It is proposed that notched ductility be used as a measure of susceptibility to notch weakening. Because values of notched ductility will vary with component geometry, it would be convenient to estimate notched-bar ductility from smooth-bar data. For design purposes, the author proposes that notched-bar reduction in area can be approximated by the reduction in area from a smooth tension test divided by the average tensile stress triaxility factor for the notch according to:

$$(\mathrm{RA}_n \simeq \frac{\mathrm{RA}}{\mathrm{TF}} \quad (3)$$

where: $(\mathrm{RA})_n$ = notched reduction in area

RA = smooth reduction in area

TF = triaxiality factor, as defined in equation (2).

Fig. 4 shows $(RA)_n$ plotted versus RA/TF for a variety of high strength steels [82] and 24ST aluminum [89] tested at room temperature as well as highly irradiated 304 stainless steel tested at temperatures ranging from room temperature to 1100°F [90]. Solid symbols represent notch weakening (NSR < 1) and open symbols represent notch strengthening (NSR > 1). The triaxiality factors for the cylindrical specimens were approximated by the Bridgman solution [78] at the centerline for the bluntly notched specimens and from the cross-section-averaged elastic solution for the sharply notched specimens. A plane stress zone was assumed for the bluntly notched thin sheet specimens with necking assumed in the thickness direction only (TF $\simeq 2.77$ [91]). It is realized that the stress calculations are not as accurate as might be desired. However, considering the large scatter in fracture ductility data, more sophisticated finite-element inelastic analyses do not appear justified for this study.

For the data plotted in Fig. 4, the relationship in equation (3) seems reasonable. Note that notch weakening occurs only for values of $(RA)_n < 20$ percent. Because the available ductility that a material possesses under the influence of a particular stress state plays an important role in determining whether the material is notch weakened or strengthened, the straight-forward relationship presented has potential in design against notch failure if it can be shown to apply in general for other materials and different notch geometries.

Proposed Criterion for Notch Weakening

As the "ductility state" of a material is reduced with high triaxiality (e.g., due to notches) or embrittling operating conditions (e.g., high energy neutron irradiation [85], corrosive environmental conditions, and cryogenic temperatures [71]) a point is reached after which it may be necessary to limit the maximum principal stress to avoid "brittle" failure. Based on the pattern of notch weakening at low values of notched ductility in Fig. 4, it is proposed that notched reduction in area be used as a measure of a material's ability to accomodate plastic flow under high levels of stress concentration and thus avoid notch weakening. To illustrate, the data in Fig. 4 are reproduced in different form in Fig. 5. The number next to each data point is the associated notch strength ratio. Although there is some scatter in the data, no notch weakening occurs for RA/TF > 20 percent. On the other hand, there is notch sensitivity for RA/TF < 10 percent, and all failures are characterized as brittle of either a transgranular cleavage or intergranular type. In the range 10 percent $<$ RA/TF < 20 percent, notch sensitivity is indeterminate.

Based on the above empirical observations, it is proposed that when RA/TF > 20 percent, no limit be imposed on the maximum principal stress provided that other stress limits to protect against different failure modes are satisfied. However, to protect against notch weakening at points of stress concentration in components subject to short-term tensile loading, it is proposed that the maximum principal stress be limited to the ultimate strength when RA/TF < 20 percent. To be conservative, it is recommended that no credit be given for compressive stress states with TF < 1. In such cases let TF $= 1$.

Discussion

Notched reduction in area appears to be a useful indicator of susceptibility to notch weakening. Its approximation from the reduction in area from a smooth tension test divided by a triaxiality factor for the notch also makes it convenient for use in design. However, most of the supporting data comes from tests conducted at room temperature and standard strain rates. Its use at elevated temperature and at high strain rates must be viewed with caution. In addition, the approximation could prove nonconservative in use with ferritic steels at a temperature close

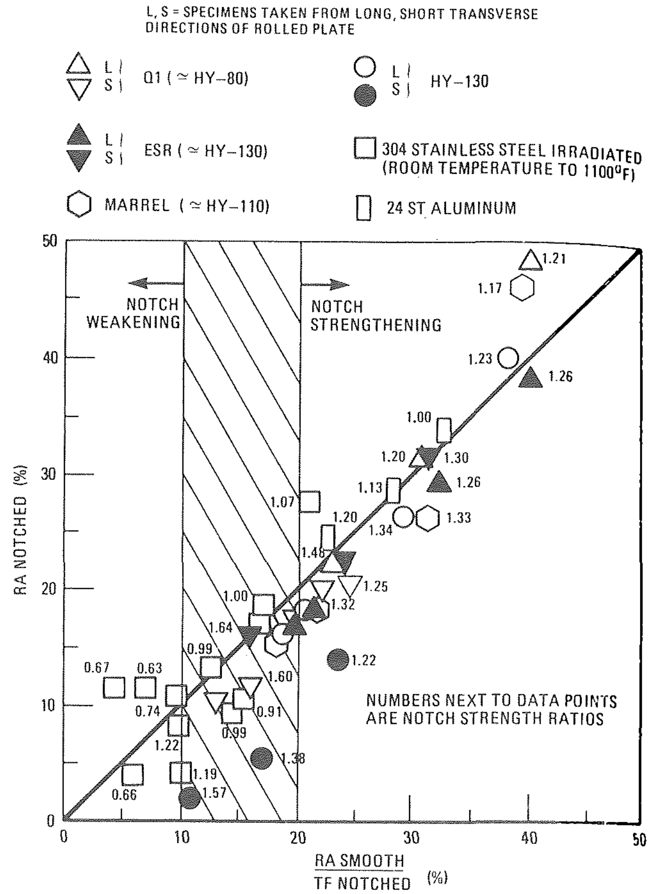

Fig. 5 Reduction in area of notched specimens versus reduction in area of corresponding smooth tension specimens divided by triaxiality factor for notched specimens

to the transition temperature. Because triaxiality is reported to raise the transition temperature of metals, the use of smooth-bar reduction in area in the transition region may not adequately represent the ductility of a material subject to triaxial tension even with the recommended triaxiality correction factor.

In limiting the maximum principal stress to the ultimate strength, the proposed criterion for notch weakening may severely limit the allowable nominal stress in components with sharp notches and high stress concentrations. The intent is to discourage designs with sharp notches in materials of limited ductility. However, this criterion may be unduly conservative for metals less sensitive to stress state than those considered in Figs. 4 and 5.

Conclusions

(1) The current study shows that notched-bar reduction in area can be approximated by reduction in area from a smooth tension test divided by a tensile stress triaxiality factor for the notch.

(2) Based on data from notched tension tests for a variety of high strength steels (including highly irradiated AISI 304 stainless steel) and one aluminum alloy, no notch sensitivity occurs for values of RA/TF > 20 percent.

(3) The results of the study suggest that a material may exhibit "brittle" behavior for values of notched reduction in area less than 20 percent and that the maximum principal stress at

the notch root be limited to the ultimate strength of the material to protect against notch weakening.

Acknowledgment

Research leading to this paper was sponsored by the U.S. Department of Energy as part of a program to develop design criteria for breeder reactor core components. The author wishes to thank M. M. Abo-El-Ata[2] and D. V. Nelson[3] for their critical review of this paper. In addition, the author is particularly grateful to D. V. Nelson for his support, encouragement, and recommendations concerning this study.

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