FLIGHT REGIME MAPPING FOR AIRCRAFT ENGINE FAULT DIAGNOSIS

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***Abstract**: One of the issues that impair the performance of aircraft engine fault diagnosis is the flight regime. When an aircraft travels from one point to another in flight regime, engine performance parameters that are used for fault diagnosing change and such changes mask the parameter changes caused by engine faults, thus make the engine fault diagnosis much more difficult. Properly addressing the flight regime issue is the key in achieving good aircraft engine fault diagnosis. Currently, the flight regime issue is typically addressed by flight regime partitioning. That is, the flight regime is partitioned into several smaller regions and each of the regions is assigned a classifier that is appropriate just for that region. There are several drawbacks associated with this approach. The fundamental one is that it requires the design and implementation of a large number of classifiers, which result in a significant increase of the costs and complexity. In this paper, a novel approach – flight regime mapping - is introduced for tackling the flight regime issue in aircraft engine fault diagnosis. The proposed flight regime mapping essentially compensates for flight regime induced parameter changes, thus accentuates the engine condition related changes, by mapping the engine parameter values from the actual flight regime to sea level static equivalent. The mapping enables classifiers that are designed for the sea level static condition to work over the entire flight regime without using multiple classifiers for different regions of flight regime. More importantly, the mapping is able to improve the performance of aircraft engine fault diagnosis. Empirical studies are conducted to demonstrate the effectiveness of the flight regime mapping approach in tackling the flight regime issue in the design of aircraft engine fault diagnostic systems.*

Keywords: Aircraft engines; Classification; Diagnostics; Flight regime; Mapping; Neural networks;

1. **Introduction:** Aircraft engine fault diagnostic (AEFD) systems are the core of the modern condition-based maintenance strategy for aircraft engines. The benefits of AEFD systems may include the following aspects [1]:

- Increasing flight safety by early detection of engine malfunctions.
- Preventing costly component damage and/or catastrophic failure.
- Reducing turnaround time by providing maintenance personnel with information on fault locations (by reducing time for manual fault isolation).
- Reducing delays and cancellations by facilitating more on-wing maintenance.
- Increasing engine on-wing time by minimizing scheduled and unscheduled engine removal.

Aircraft engine fault diagnosis is a difficult task due to the following intrinsic characteristics related to aircraft engines:

- There exist engine initial quality variations. Engine initial quality variation is the results of the variation of fabricating and assembling. The engine initial quality varies from engine to engine even within the same engine models.

- Engine quality deteriorates over time. Engine deterioration is caused by many effects, such as, tip clearance changes in the rotating components, seal wear, blade fouling, blade erosion, blade warping, foreign object damage, actuator wear, and blocked fuel nozzles [2]. Engine deterioration results in engine performance parameter changes over time (time-varying).

- Aircraft engines are operated at different points in flight regime. When an aircraft travels from one point to another in flight regime, the engine performance parameters change following the principles of thermodynamics and aerodynamics.

Due to the importance and the challenges, aircraft engine fault diagnosis has drawn tremendous amount of research interests. With advances in modern aircraft engines, designing a reliable and cost-effective AEFD system continues to be the most interesting research topic in fault diagnosis.

This paper is concerned with the flight regime issue in the design of AEFD system. Engines when operating at different points in flight regime result in changes of engine performance parameters. Such "inherent" or not-fault-related changes may resemble the changes caused by engine faults/abnormality and therefore "confuse" the engine fault diagnostic system. Properly addressing such flight regime issue is the key to improving the performance of AEFD systems. In this paper, we propose an innovative method, flight regime mapping, to tackle the flight regime issues so that a more accurate and reliable AEFD system can be achieved.

The rest of this paper is organized as follows. Section 2 presents the flight regime problem and the related work in addressing the flight regime problem. Section 3 details proposed flight regime mapping method. Section 4 describes an AEFD system design example that is used to demonstrate the effectiveness of flight regime mapping. The classification performance of the AEFD system designed using flight regime mapping is given in Section 5. Section 6 concludes the paper.

2. Flight regime problem and related work: One of the inherent characteristics of aircraft engines is that they need to be operated at various points of flight regime (different altitudes and Mach numbers), more broadly at various operating points in the operating space typically defined by four dimensions: the altitude, the Mach number, the ambient temperature, and the TRA value (the power setting). When aircraft travels from one point to another in flight regime, or from one operating point to another, the engine performance parameters that are used for fault diagnosing change and such changes mask the parameter changes caused by engine faults, thus makes the engine fault diagnosis much more difficult. To illustrate this masking effect, let's look at

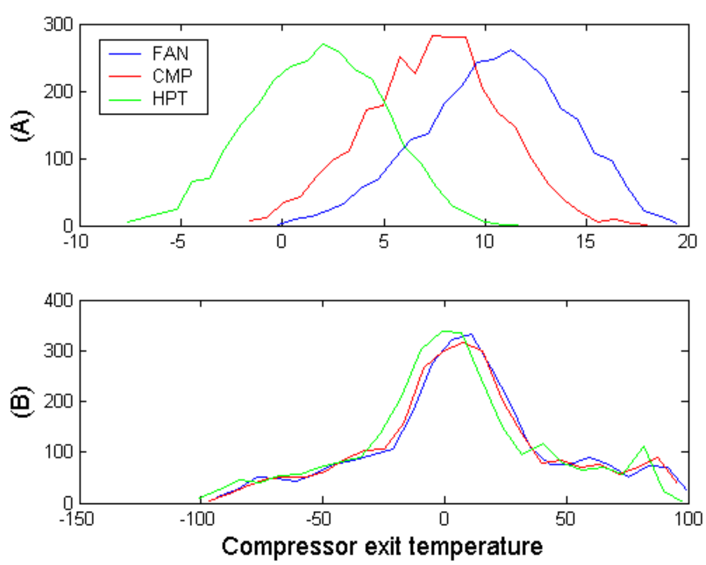


Figure 1: Flight regime effects on engine parameters

Figure 1, where histograms of the compressor exit temperature, one of the engine performance parameters, for three engine conditions are shown.

While Figure 1.A shows the compressor exit temperature at sea level static (S.L.S), Figure 1.B is for the same engine parameter over entire flight regime. Under S.L.S. condition (Figure 1.A), the difference of this engine parameter among the three engine conditions is noticeable although some overlapping does exist. The variation of this engine parameter within each engine condition under SLS condition is due to the engine-to-engine variation and different levels of engine deterioration. However, over entire flight regime (Figure 1.B), the histograms of this engine parameter for three engine conditions are almost completely overlapped, i.e., inseparable among the three engine conditions, due to the significantly increased variation resulted from flight regime. From Figure 1, we can see that flight regime causes a significant amount of reduction in class separability. As a result, most aircraft engine fault diagnostic systems that perform reasonably well within a small region in flight regime usually perform poorly over entire flight regime.

Currently, the flight regime issues are typically addressed by flight regime partitioning. That is, the entire flight regime is partitioned into several smaller regions. Each of the regions is assigned a classifier that is trained just for that region. A scheduling algorithm is used to switch between appropriate classifiers based on the operating condition. For example, this approach was used in the study of Mast et al [3] for aircraft engine fault identification and in the study of Embrechts et al [4] for turbofan engine parameter modeling. This flight regime partitioning approach is conceptually simple and intuitive. However, it bears the following drawbacks:

- The overall performance of the fault diagnostics system designed depends on how the flight regime is partitioned. Since there is no systematic method to guide the partitioning, the partitioning is usually empirical. As a result, the number of sub-regions tends to be large, e.g., 63 models were used in Embrechts' work, which demands significant resource in terms of design time and efforts, and that in turn increases the design costs.
- The partition boundary is crisp. Two similar operating conditions that are represented by two adjacent points in the flight regime may be assigned to different classifiers simply because the two points are located in different sides of the boundary. As a result, classification discontinuity may occur in the neighborhood of the partition boundary.

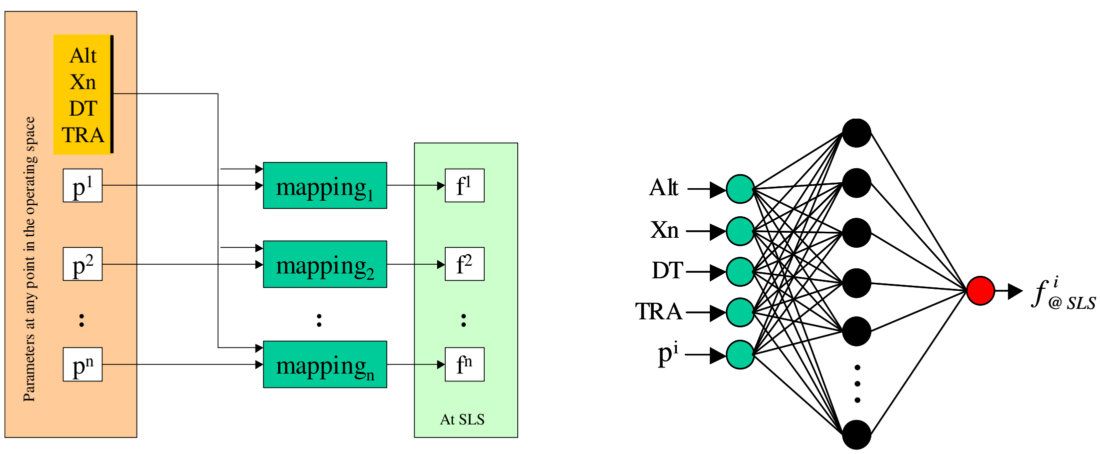
3. Flight regime mapping using neural networks: In this paper, the flight regime issue encountered in design of aircraft engine fault diagnosis is tackled through an innovative approach – flight regime mapping. As discussed before, engine performance parameters change as engine travels from one point to another in flight regime. It has also been illustrated in Section 2 that such flight regime induced parameter changes mask the parameter changes caused by engine faults, thus greatly increase the difficulties of engine fault diagnosis. Imaging somehow we can eliminate the flight regime induced disturbance from engine parameter values and design the AEFD system based on these corrected parameters, we would expect an improved performance in terms of accuracy and reliability of the AEFD system. Flight regime mapping proposed in this paper does just that.

Flight regime mapping is essentially to map engine performance parameters from the actual flight regime to a common point (e.g., sea level static) in the engine operating space. At this common point, any engine parameter differences will be primarily due to engine faults, thus results in a higher AEFD performance.

The mapping idea is natural and intuitive since engines have to follow physics laws of thermodynamics and aerodynamics over entire flight regime all the time. The physics laws constitute a fixed functional relation of engine performance parameter values between any two points in flight regime if engine condition is assumed unchanged. Obviously for a real engine, however, such functional relation will be highly nonlinear and complex. Explicitly expressing the functional relation would almost be impossible considering the nonlinear and dynamic nature of engine and noisy environment. In this paper, the use of neural networks to represent the functional relation for flight regime mapping is investigated. Neural networks are universal function approximators that can approximate every bounded continuous function with arbitrarily small error [5 & 6]. The data-driven character makes neural networks more powerful than other methods for complex function relation identification.

Figure 2 (a) illustrates the architecture of flight regime mapping. To reduce the model complexity and to increase the mapping accuracy as well, one-map-per-parameter mapping scheme is used. Each mapping takes inputs including the four engine operating space descriptors (altitude, Mach number, ambient temperature, and TRA) and the specific engine performance parameter to be mapped. The mapping targets are the engine performance parameter values *for the same engine condition*, but at sea level static (SLS). That is, the mapping outputs the equivalent engine parameter values at SLS. Mathematically, the mapping for i^{th} parameter p^i can be expressed as:

$$f^i_{@SLS} = F_i(Alt, Xn, DT, TRA, p^i) \qquad (1)$$



(a) **(b)**

Figure 2: (a) Flight regime mapping architecture; (b) Flight regime mapping NN structure

Neural network structure that represents this mapping function is shown in Figure 2 (b). The network is a two-layer (one hidden layer) feed-forward type and has sigmoid activation functions in the hidden layer and linear transfer functions in the output layer.

The flight regime mapping seems to be a fairly straightforward concept for dealing with the flight regime problem. However, to make it work effectively, there are issues to be resolved. Take a closer look at the mapping function shown in Equation 1. It may resemble a typical prediction function, where Alt, Xn, DT, TRA, and p^i can be thought of as the independent variables while $f^i_{@SLS}$ is the dependent variable. However, there is a unique characteristic associated with flight regime mapping. For a given point in the operating space, i.e., with Alt, Xn, DT, and TRA being a constant, the engine parameter p^i varies with three engine-related factors: 1) the engine health condition (i.e., normal or faulty), 2) the engine initial quality, and 3) the engine deterioration level. These three factors do not explicitly appear in the mapping function as part of independent variable. This is because in real applications one has no knowledge of the three factors, especially, engine health condition (otherwise we wouldn't need a fault diagnostic system) and engine initial quality. From the standpoint of engine fault diagnosis, since it relies on the engine parameter changes caused by engine conditions (factor 1 above), it requires the mapping to maximally preserve this portion of difference/variation while eliminating flight regime effects so that the AEFD system designed based on the mapped parameters has good classification performance. To meet this requirement, the dependence of the engine parameter, p^i, on the last two factors (engine initial quality and engine deterioration level) that can be thought of as noise or variation of the independent variable in a typical prediction function has to be somehow largely eliminated, that is, to take the 2 factors out of the mapping function. This can only be done if the engine initial quality and engine deterioration level are known. Uncertainty of these two factors, on the other hand, will lead to a less accurate mapping, and thus a less improvement in classification performance. In this paper, we assume that engine deterioration can be estimated based on engine deterioration rate and engine service hours, that is, engine deterioration level is known during mapping.

4. An AEFD design example: To demonstrate the effectiveness of the proposed flight regime mapping method in improving the performance of AEFD system, a real-world AEFD system is designed. The AEFD system concerned in this paper is a fault diagnostic system of commercial aircraft engines, which is designed for detecting & diagnosing six *engine gas path faults.* They are: 1) fan blade damage (FAN); 2) compressor blade damage (CMP); 3) high pressure turbine fault (HPT); 4) low pressure turbine fault (LPT); 5) customer discharge pressure leakage (CDP); and 6) variable bleed value fault (VBV).

4.1. Design data: Since well-distributed data, especially for faulty engines, are difficult to obtain from real engines, AEFD system design nowadays still primarily relies on simulation to generate engine parameter data for different engine conditions. For data generation in this study, an engine simulation that uses a real-time, nonlinear engine model together with FADEC is used. The same simulation scheme was used for data generation in the recently completed IMATE project [7].

The engine operating space covered by the simulation consists of three flight phases, i.e., ground idle, takeoff, and cruise, are considered The simulated engine is operated at five different engine deterioration levels, namely, 0%, 25%, 50%, 75%, and 100%, where 0% means no deterioration (new engine) and 100% means engine has reaches the end of its service life. Additionally, the simulation also takes into account of the random variations including sensor bias and engine initial quality as well as ambient temperature variations to encompass hot and cold days operation.

The simulation yields a total of 19,635 data points that are evenly distributed in seven engine condition classes (one normal engine class and six faulty engine classes).

4.2. Data preprocessing: For a typical fault diagnosis system design, preprocessing the data is almost always necessary. Data preprocessing is not just a step that simply formats data properly so that the downstream design processes (feature selection and classifier design) can be implemented. Proper data preprocessing, in fact, can effectively improve the performance of the diagnostic system designed. In that sense, data preprocessing is an important sub-task in fault diagnosis design. Typical data preprocessing include steps like data formatting, data scrubbing, and missing data handling, etc. In this study, since the data is generated from the simulation, data formatting and cleaning become unnecessary. Our focus, hence, is on TRA effects removal and data normalization. Our preliminary study shows both processes are effective in improving the performance of the AEFD system.

After examining the engine performance parameters through visualization, we observed that the values of the sensed engine performance parameters are highly correlated with the throttle resolver angle (TRA) values. To illustrate the correlation, the "compressor exit pressure" parameter, as an example, is plotted against TRAs in Figure 3.a, where different colors represent data points from different engine condition classes. It is clear that the compressor exit pressure value increases almost linearly with the TRAs.

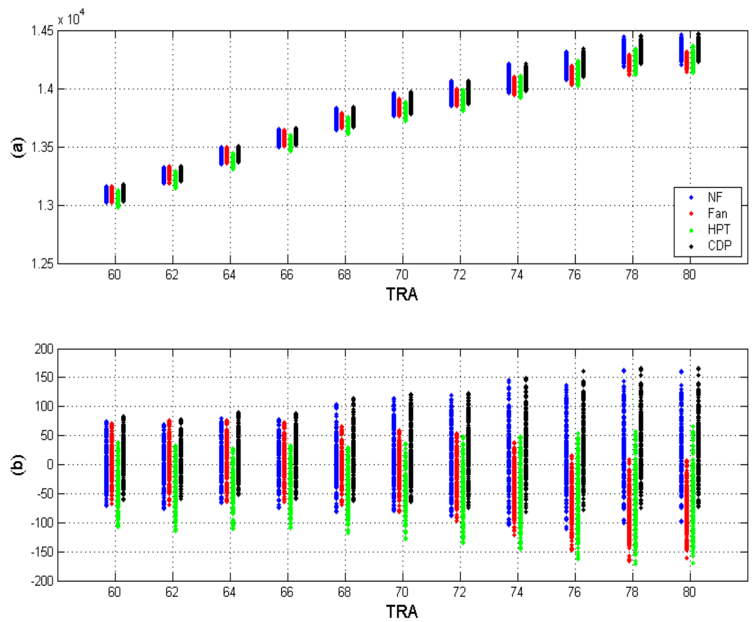


Figure 3: Compressor exit pressure –vs- TRA:
(a) Before TRA removal; (b) After TRA removal

The strong dependence of the sensed parameters on the TRA degrades the classifier performance. From Figure 3.a one can see that different engine faults do introduce certain noticeable changes in compressor exit pressure values (different color bars in each cluster in Figure 3.a). However, changes of the compressor exit pressure due to different levels of TRAs are much more significant (from one cluster to another in Figure 3.a). Significant changes of the parameter values due to TRA "mask" the subtle changes of the parameter values introduced by engine faults. As a result, diagnosing the engine faults becomes more difficult since the degree of difficulty of a classification task increases as the magnitude of the within-class variability increases with respect to among-class differences [8].

To remove the TRA effects, the mean of the feature values for all classes at each TRA level are subtracted from the feature values. Graphically, this is to bring the center of each cluster of data in Figure 3.a to the zero level.

The compressor exit pressure is again plotted against TRAs in Figure 3.b after the TRA effects are removed. Comparing Figure 3.b with Figure 3.a, it is clear that removing the TRA effects accentuates the difference of the feature values between classes, i.e., more separable.

Normalization for pattern classification problems is typically to scale all features to a common range so that effects due to arbitrary feature representation (e.g., different units) can be eliminated

[8]. There are different methods for normalization. In this paper, the range normalization, the most common one, is used.

5. Results: Followings are the results of the flight regime mapping and of the classification for the AEFD systems concerned.

5.1. Flight regime mapping: Each of the mapping neural networks is a 3-layer feed-forward network, i.e., is of [5, 15, 1] structure. The network is trained using the Levenberg-Marquardt learning algorithm.

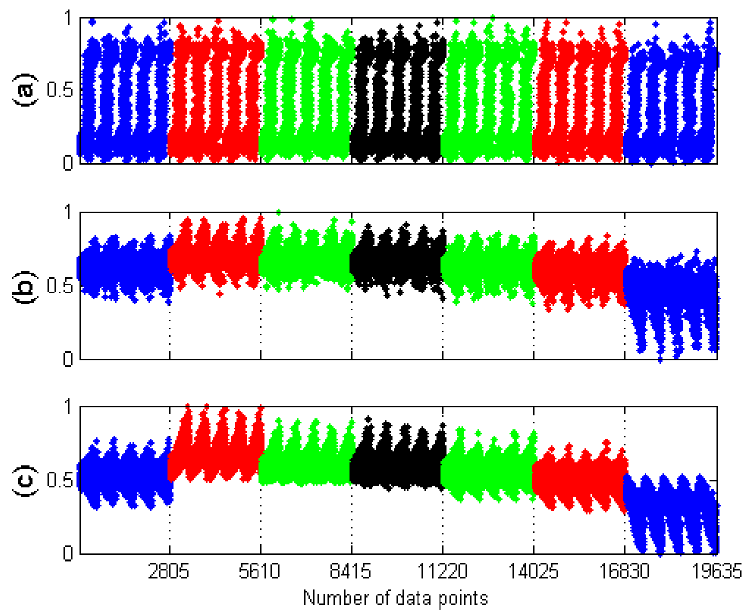


Figure 4: Mapping results

To show the effectiveness of the mapping, the mapped engine parameters are compared with those before mapping and those at SLS (the mapping targets).

Figure 4 illustrates such comparison for compressor exit pressure - one of the sensed engine parameters. Different colors in Figure 6 represent different engine condition classes and the parameter values are normalized into a range of [0,1]. Figure 4 (a) shows the distribution of compressor exit pressure before flight regime mapping, while Figure 4 (b) shows the same parameter after flight regime mapping. By comparing Figures 4 (b) with 4 (a), one can see that flight regime mapping significantly reduces the variation of the engine parameter within each engine condition class and increases the class separability between classes, thus improves the classification performance. Figures 4 (c) shows the distribution of the same engine parameter at sea level static, which is used as the targets for training the mapping neural networks. The similarity between Figures 4 (b) and 4 (c) is significant, which indicates that the NN mapping performs reasonably well.

To mathematically quantify how well the flight regime mapping performs, the R^2 value [9], a statistical index for measuring goodness of fit is used. The R^2 has a range between 0 and 1, where 1 indicates a perfect fit and a zero means no match at all. The calculated R^2 values for all sensed engine performance parameters are shown in Table I. The numbers in Table I indicate that the NN mapping performs well in mapping the engine parameter values from different points in flight regime to SLS.

Table I - R^2 values of flight regime mapping

	Engine performance parameters								
	N1	**N2**	**PS3**	**PS13**	**P25**	**T25**	**T3**	**T495**	**T5**
R^2	0.999	0.992	0.994	0.994	0.988	0.987	0.991	0.984	0.967

5.2. Classification: The mapped engine performance parameters are then used for classifier development (training and testing). Since our focus in this paper is on the innovative flight regime mapping method, specifically on demonstrating the effectiveness of the flight regime mapping in improving classifier performance, we will not try to explore the best classification system for AEFD. Rather, we only use one type of classifier, namely neural network classifier for demonstration. It is our belief that using neural networks as a classifier can serve our purpose of demonstrating effectiveness of flight regime mapping without loss of generality. Exploring the best design of classification system for AEFD will be the topic of a separate paper.

The classifier: As we can see from section 4, the AEFD system concerned in this paper involves diagnosing 7 different engine conditions (1 normal condition and 6 different types of faults). That is, the classifier to be designed has 7 different outputs, which is typically referred as a multi-class classification problem [10]. For NN classifiers, the multi-class classification can be handled directly, i.e., to structure network such that the number of output nodes equal to the number of classes. However, studies have proved that performance can be improved if the multiclass problem is decomposed into a series of binary classification ones [11]. There are several ways to decompose the multiclass classification problem into a series of binary classification ones in order to achieve better performance [12]. In this study, we take "one-vs-other" method to decompose the AEFD classification problem into 7 binary classifiers. Each of the seven classifiers is trained to distinguish between patterns belonging to a class (C_i) and its complement ($\overline{C_i}$) (combining all data not belonging to class C_i). To classify an unknown input x, the outputs of the 7 binary classifiers form a 7-component vector and are combined to arrive at a final classification decision.

Each binary classifier is a 3-layer feed-forward neural network, where the number of input nodes is 11 (9 sensed engine parameters plus 2 manufactured features), and the output node is always one. The activation functions for all layers are of the same type, i.e., the hyperbolic tangent sigmoidal function. The network is trained using the Levenberg-Marquardt learning algorithm, which has the fastest convergence. Additionally, to prevent saturation, the target values are scaled to +0.9 for positive cases and to –0.9 for negative cases. The error on a separate validation set is monitored during the training process as a measure to stop the training, thus to prevent overfitting.

The performance indices: Three performance indices (overall accuracy, false positive rate, and false negative rate) extracted from a confusion matrix are used for classifier performance comparison/evaluation. For multiclass classification, the three performance indices are defined as follows.

Let $CM(i, j)$, $i, j = 1,...C$ be the confusion matrix, where C is the number of classes. And assume Class 1 represents normal (fault-free) engine condition.

Overall accuracy: $$OAC = \sum_{i=1}^{C} CM(i,i) \Big/ \sum_{i,j=1}^{C} CM(i,j) \quad (2)$$

False positive rate: $$FPR = \sum_{j=2}^{C} CM(1,j) \Big/ \sum_{j=1}^{C} CM(1,j) \quad (3)$$

False negative rate: $$FNR = \sum_{i=2}^{C} CM(i,1) \Big/ \sum_{i=2,j=1}^{C} CM(i,j) \quad (4)$$

The classifier is evaluated by 5-fold stratified cross-validation [13]. The classification results shown in this section are actually the average of the classification results of the 5-fold cross-validation.

The results: To demonstrate the effectiveness of flight regime mapping in improving classification performance, the classification results of the AEFD system concerned are shown here for two different designs. They are: 1) the baseline design, where the original engine performance values over all flight regimes are directly used for designing the AEFD; and 2) the design with flight regime mapping, under which the parameter values are first mapped from actual flight regime onto sea level static (SLS) before they are used for classifier design. The flight regime mapping here is designed with engine deterioration level being assumed known (see section 3 for discussion)

The confusion matrices and the extracted performance indices for the two designs are listed in Tables II and III, respectively.

From Tables II and III, we can see that flight regime mapping (Table III) increases the overall accuracy by more than 13 percentage points, reduces the false positive error by more than 17 percentage points, and reduces the false negative error by approximately 7 percentage points, comparing to the those from baseline design (Table II).

It is worthwhile to point out that the relatively low performance of the three designs shown in this section is due to the facts of 1) the classifier used is not the optimal and 2) the AEFD system design is a model-free method, which typically shows lower performance than model-based methods. However, model-free method deprives the model-based method of the requirement of an accurate engine model, which is difficult to obtain in real-world applications.

Table II: Classification results for baseline design

		Predicted Classes							Per Class	Performance
		NF	FAN	CMP	HPT	LPT	CDP	VBV	Accuracy	Indices
True Classes	NF	1656	7	195	90	620	182	55	59.04	
	FAN	2	2777	13	11	0	1	1	99.00	
	CMP	475	84	1678	98	347	110	13	59.82	OAC= 70.39
	HPT	317	103	67	1733	527	25	33	61.78	FPR= 40.96
	LPT	625	22	98	311	1547	180	22	55.15	FNR= 12.01
	CDP	546	11	129	8	323	1749	39	62.35	
	VBV	57	5	9	3	28	21	2682	95.61	

Table III: Classification results for design with flight regime mapping

		Predicted Classes							Per Class	Performance
		NF	FAN	CMP	HPT	LPT	CDP	VBV	Accuracy	Indices
True Classes	NF	2139	7	106	150	197	70	136	76.26	
	FAN	0	2786	13	0	2	0	4	99.32	
	CMP	118	7	2454	62	94	57	13	87.49	OAC= 83.65
	HPT	188	2	76	2210	291	16	22	78.79	FPR= 23.74
	LPT	288	2	81	317	1893	177	47	67.49	FNR= 5.19
	CDP	202	1	86	7	190	2292	27	81.71	
	VBV	77	1	4	8	37	28	2650	94.47	

6. Conclusions: Several issues make aircraft engine fault diagnosis one of the most difficult diagnostic problems. One of such issues is related to flight regime. When an aircraft travels from one point to another in flight regime, engine performance parameters that are used for fault diagnosing change and such changes mask the parameter changes caused by engine faults, thus make the engine fault diagnosis much more difficult. To tackle the flight regime issues, an innovative flight regime mapping using neural networks is proposed in this paper. The mapping reduces the disturbance caused by change of flight regime to engine performance parameter changes and thus accentuates the engine fault induced parameter changes. As the result, classifiers designed based on the mapped engine performance parameters yield a much better performance. In this paper, we have demonstrated the effectiveness of the flight regime mapping in improving classification performance of AEFD through designing a real-world AEFD system.

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