

# Tensegrity Inverse Kinematics: The Force-Density Method

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September 23, 2016

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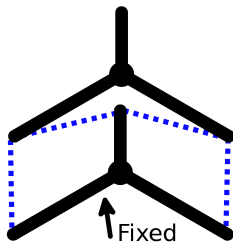
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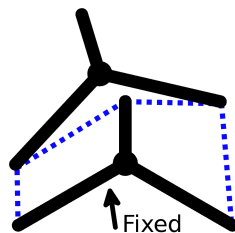
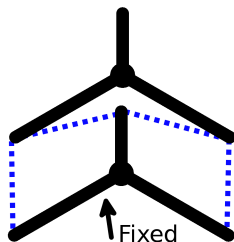
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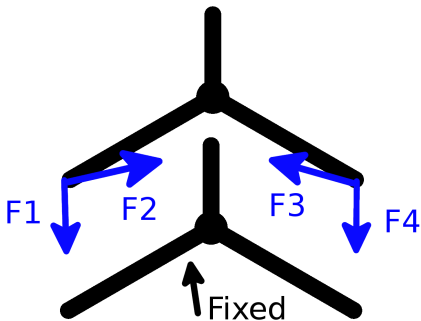
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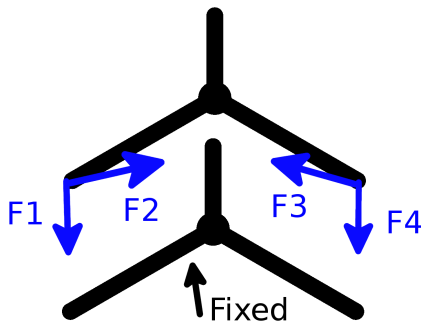


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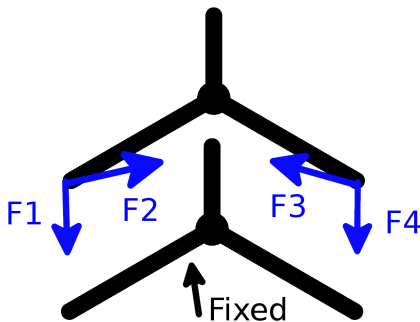
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## Assumptions?

At each position of the rods...

$$\sum_{(cables)} F_{cable} = 0$$

...instead of:  $\sum_{(cables)} F_{cable} = ma$



*At each position in the trajectory, the rods are not moving.*

# The Inverse Kinematics Problem

Given rod positions, find cable rest lengths.

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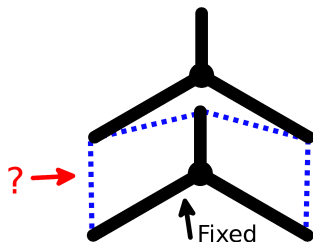
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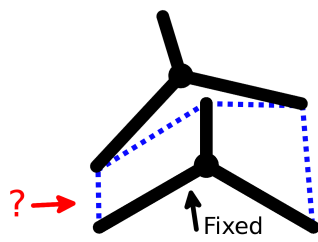
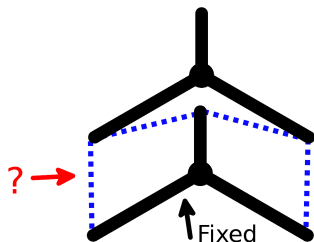
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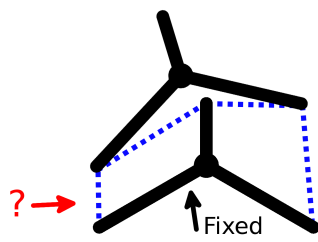
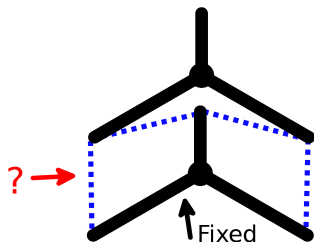




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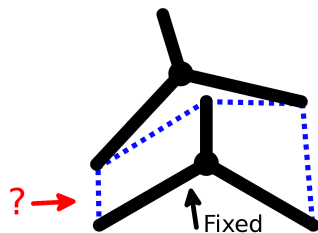
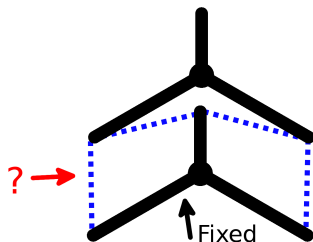


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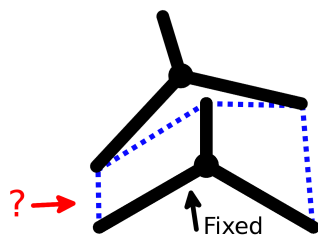
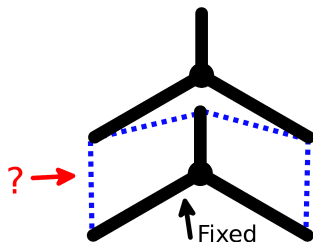


Remember:  $F_{(cable)} = \text{spring force}$

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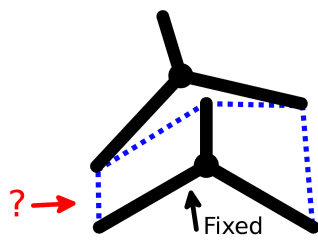
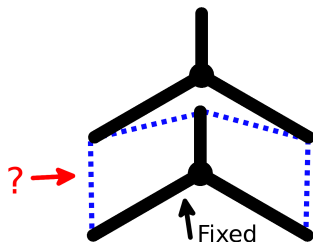


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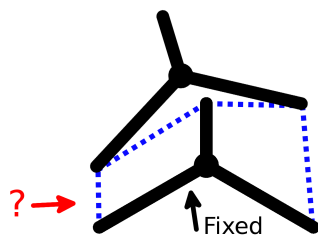
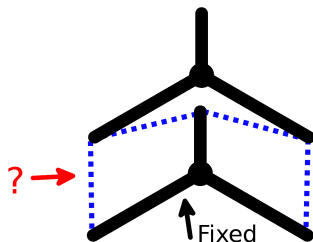
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$$F_{(cable)} = k(L - L_0)$$

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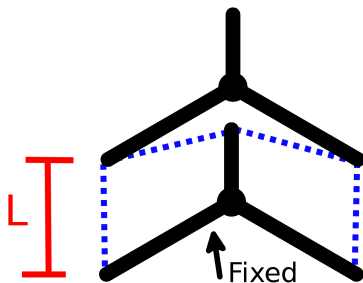
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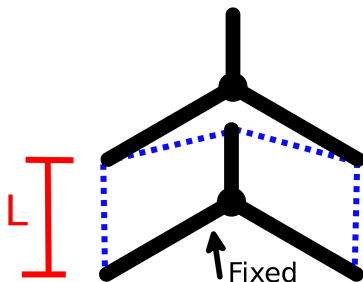
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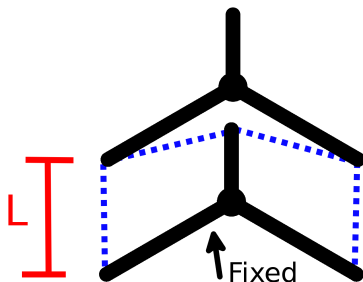
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Given rod positions, we can always calculate  $L$ , so finding forces is equivalent to finding rest lengths.

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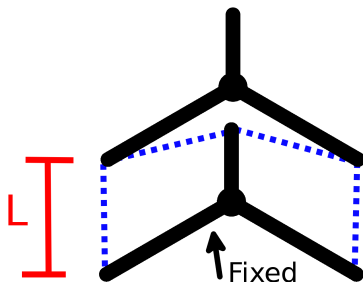
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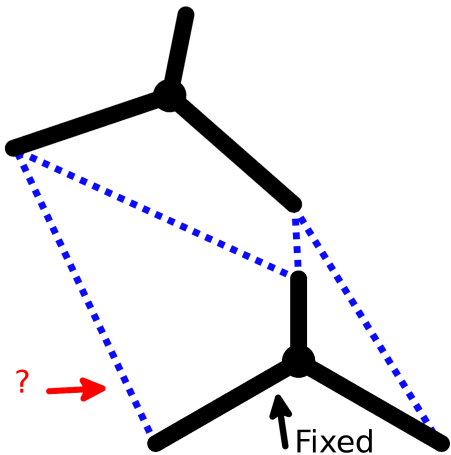
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# Does this problem always have a solution?

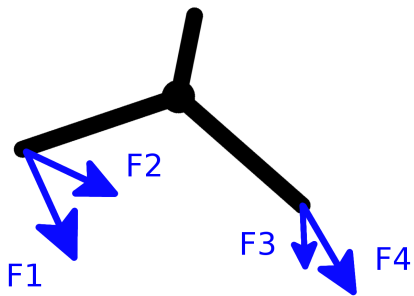
Are there always a set of cable lengths such that  $\sum F = 0$  ?

# What about this location of the rods?

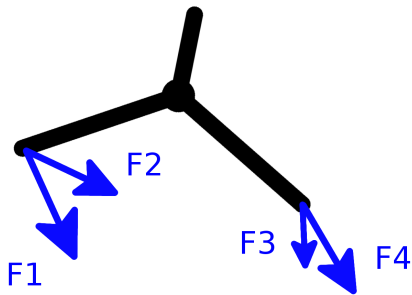
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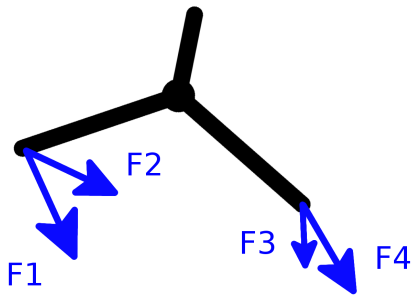


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Think: is it possible for  
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What about this location of the rods?



Think: is it possible for  
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...not possible!

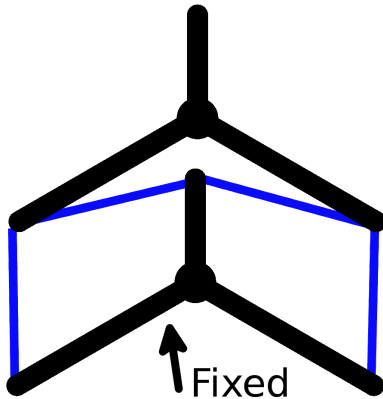
# Does a solution always exist?

NO.

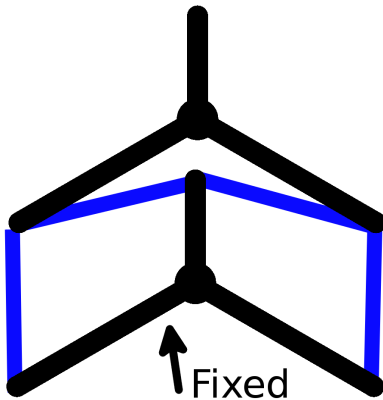
*Sometimes, no solution exists!*



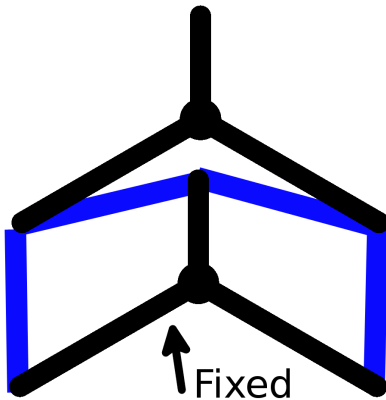
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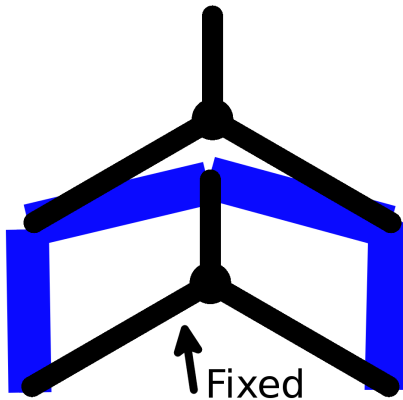
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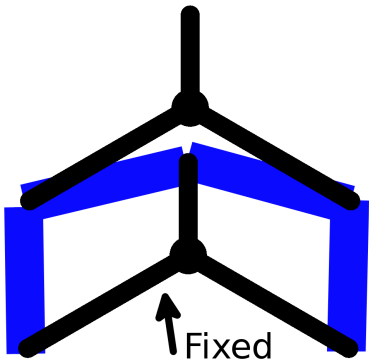
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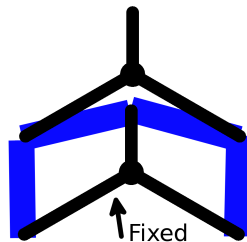
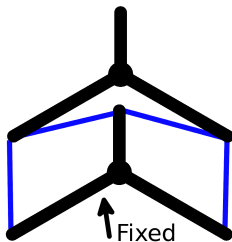
## How many solutions exist?

The cables can always be tightened more!

If one solution exists,  
then  $\infty$  solutions exist!

# Which solution to choose?

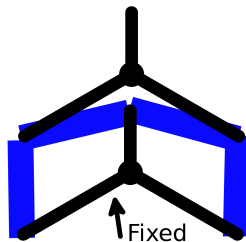
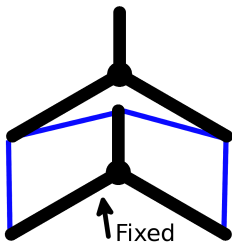
...be kind to our motors.





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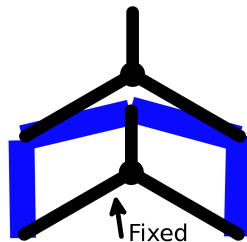
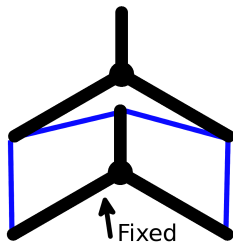
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# Which solution to choose?

...be kind to our motors.



Choose the one with the thinnest lines!

Choose the solution with the lowest *force density*.

# The solution to the force-density method

$$\begin{aligned} & \underset{\mathbf{w}}{\text{minimize}} && \sum_{(cables)} (\text{force density}) \\ & \text{subject to} && F_{(cable)} \geq 0 \end{aligned}$$

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$$q = A\mathbf{p} - B\mathbf{w}$$

$$\underset{\mathbf{w}}{\text{minimize}} \quad \mathbf{w}^T B^T B \mathbf{w} + 2\mathbf{w}^T B^T A \mathbf{p}$$

$$\text{subject to} \quad A\mathbf{p} + B\mathbf{w} \geq 0$$

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This is a *quadratic program*: it can be solved easily in MATLAB.

Finally, to get  $L_0$ ,

$$L_0 = L - \frac{L \times q}{k}$$

# Videos!

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  - ...using “quadprog” in MATLAB
  - ...example on the ultra-spine-simulations GitHub repository.

# Further Reading I



J. Friesen, A. Pogue, T. Bewley, M. de Oliveira, R. Skelton, V. SunSpiral.

DuCTT: A tensegrity robot for exploring duct systems  
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## Further Reading II



A.P. Sabelhaus, H. Ji, P. Hylton, Y. Madaan, C. Yang,  
A.M. Agogino, J. Friesen, V. SunSpiral.

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