Tensegrity Inverse Kinematics: The Force-Density Method

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University of California, Berkeley

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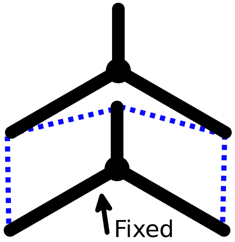

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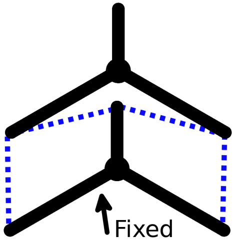

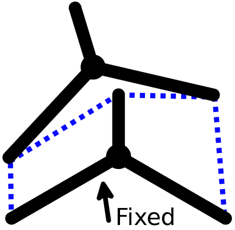

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At each position of the rods...

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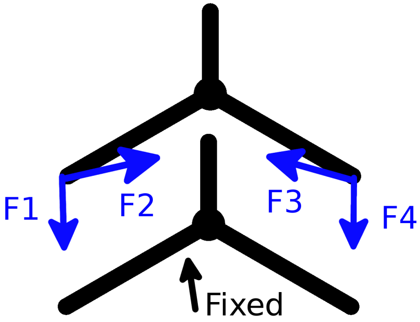

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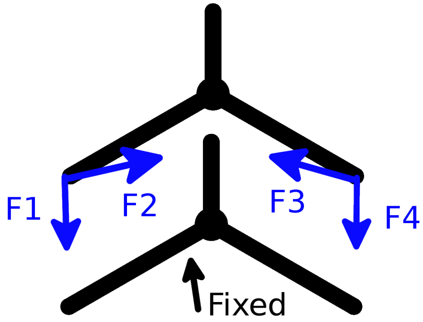

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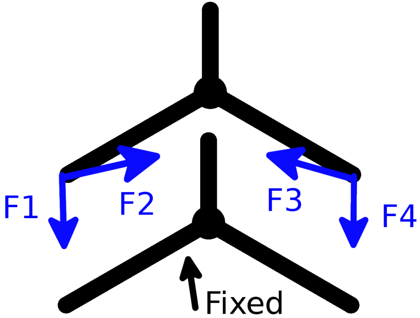

At each position in the trajectory,
the rods are not moving.

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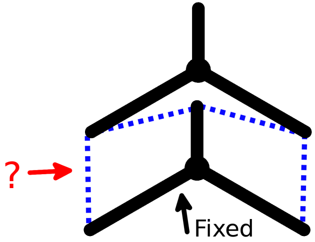

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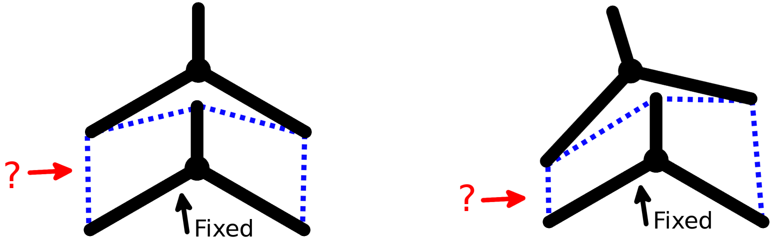

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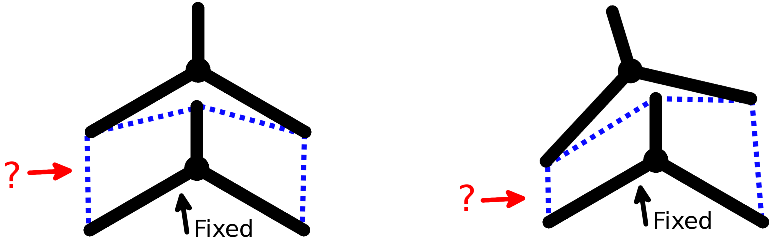

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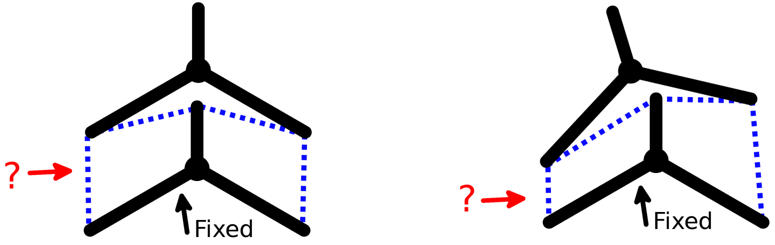

Remember: $F_{(cable)} = \text{spring force}$

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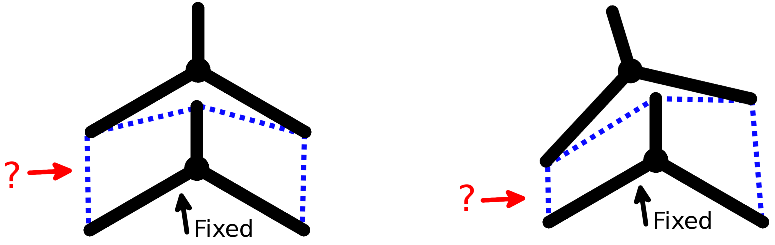

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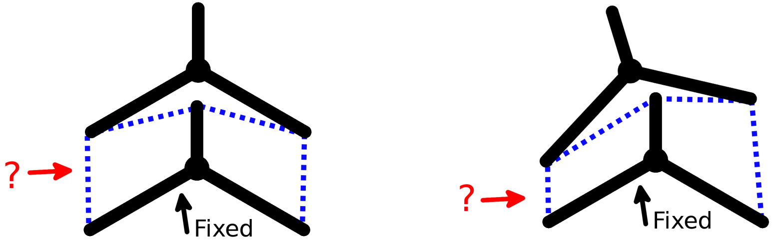

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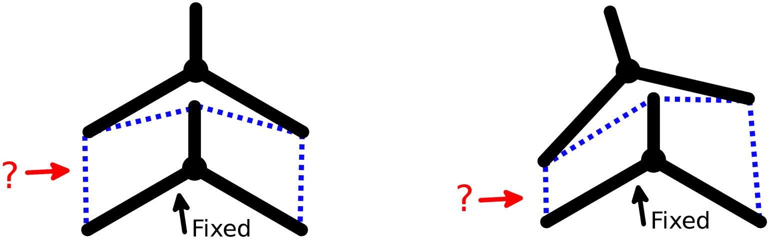

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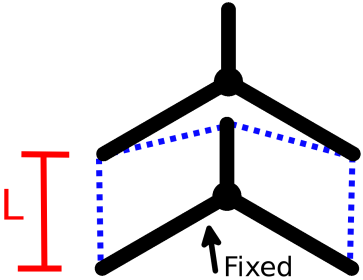

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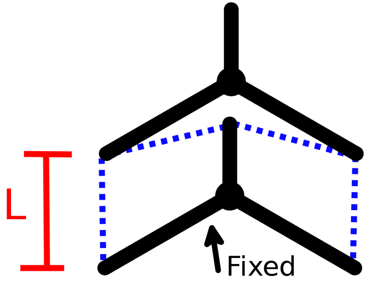
The Inverse Kinematics Problem



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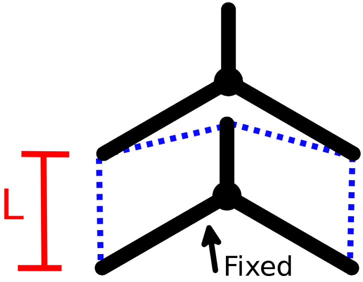

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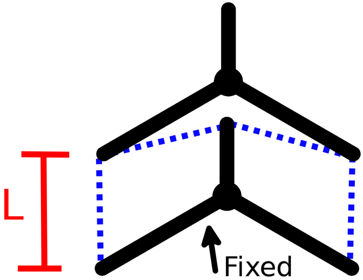

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Does this problem always have a solution?

Are there always a set of cable lengths such that $\sum F = 0$?

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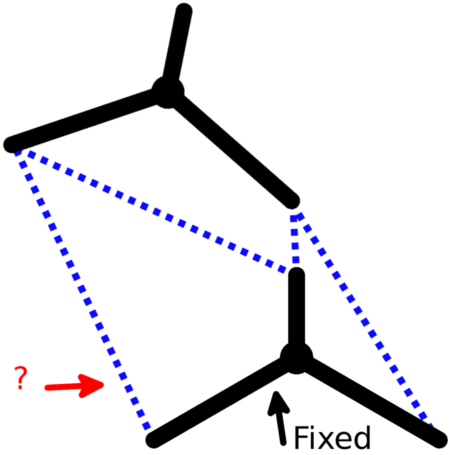

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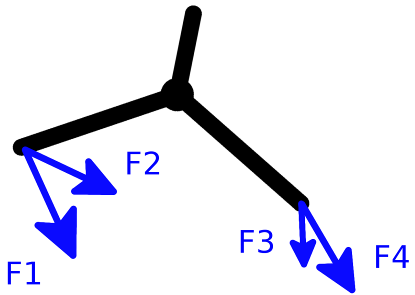

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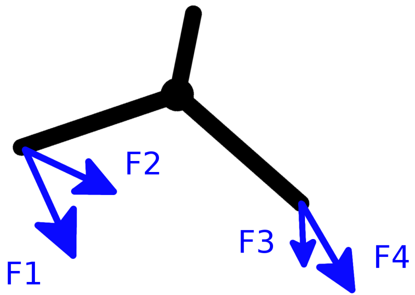

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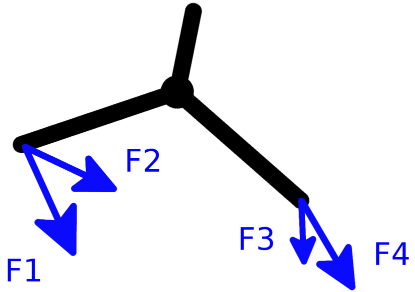
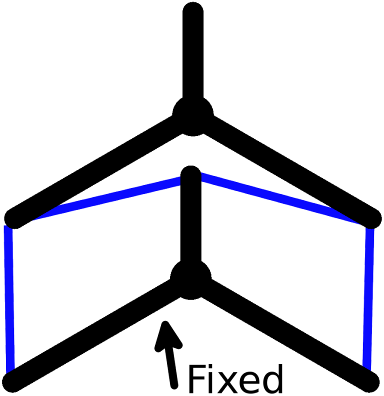


Think: is it possible for
$\sum F_y = 0$??
...not possible!

Does a solution always exist?

NO.
Sometimes, no solution exists!

How many solutions exist?



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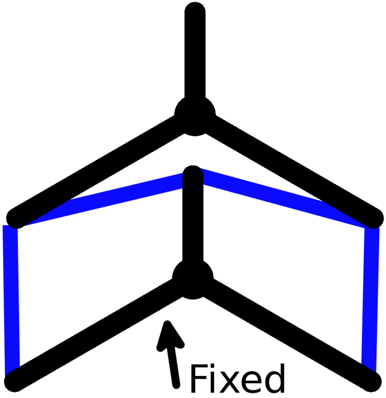

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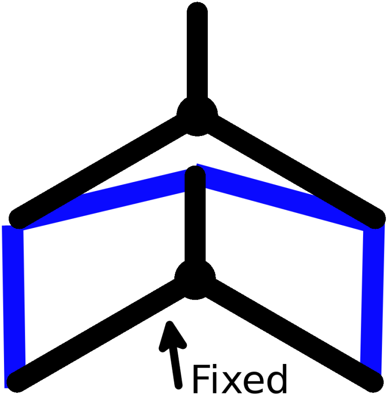

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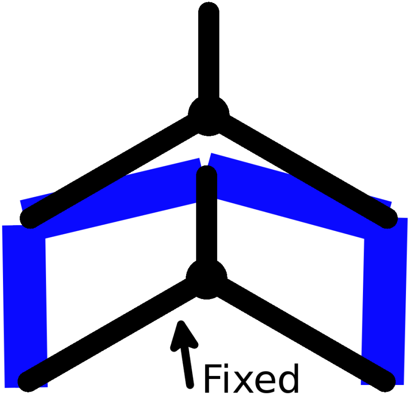

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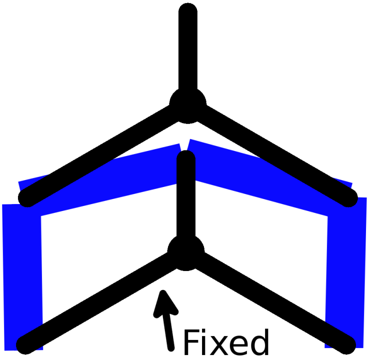

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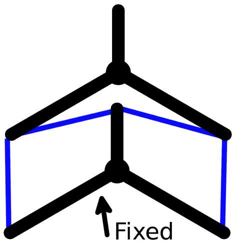
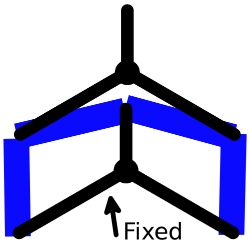
If one solution exists, then ∞ solutions exist!

Which solution to choose?

...be kind to our motors.





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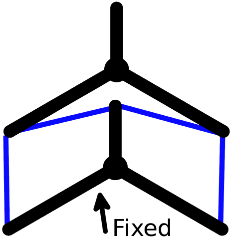

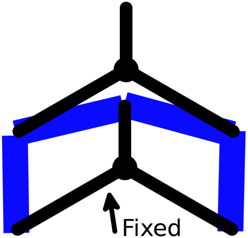

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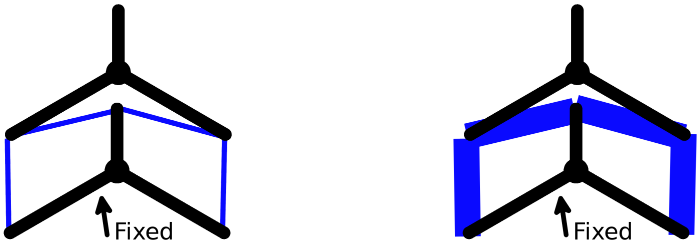

Choose the one with the thinnest lines!

Choose the solution with the lowest *force density*.

The solution to the force-density method

$$\begin{aligned} \underset{\boldsymbol{w}}{\text{minimize}} \quad & \sum_{(cables)} (\text{force density}) \\ \text{subject to} \quad & F_{(cable)}) \geq 0 \end{aligned}$$

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This is a *quadratic program:* it can be solved easily in MATLAB.

Finally, to get L_0,

$$L_0 = L - \frac{L \times q}{k}$$

Videos!

Conclusion

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  - ...using “quadprog” in MATLAB
  - ...example on the ultra-spine-simulations GitHub repository.

Further Reading I

J. Friesen, A. Pogue, T. Bewley, M. de Oliveira, R. Skelton, V. SunSpiral.
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IEEE International Conference on Robotics and Automation (ICRA), 2014.

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Further Reading II

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