RAPID AND AGILE LOCOMOTION WITH POWER-DENSE MILLIROBOTS

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MOTIVATION | DESIGN FOR ENERGETIC LOCOMOTION

Try to design highest performance robots

Use those robots as scientific tools to study locomotion
To perform better at energetic tasks, design with power density as primary criteria.

Method: Use one actuator to do all the energetic work. This is enabled by a coupling mechanism that produces the desired behavior by default, when power is applied.
APPROACH | DESIGN FOR POWER-DENSITY

Test power density first design with running robot and jumping robot.

Both should be best in class

X2-VelociRoACH

Salto

150 mm

10.4 cm
# OVERVIEW

RAPID AND AGILE LOCOMOTION WITH POWER-DENSE MILLIROBOTS

## 1: RAPID LOCOMOTION
- Approach
- Result
- Stride kinematics
- Force and power
- Energetics

## 2: AGILE LOCOMOTION
- Motivation
- Agility metric
- Actuation strategies
- SE+MA actuation
- Leg mechanism
- Proof of concept
- Prototype & results

## 3: SPATIAL-AGILE
- Attitude control
- Spatial controller
- In-place jumping
- Forwards-backwards
- Jump exploration
1: RAPID LOCOMOTION

Approach
Result
Stride kinematics
Force and power
Energetics
1: RAPID LOCOMOTION

Approach
Find the speed limits of legged locomotion

What limits the top speed of a robot?
Design for power-density first:
Cut everything that doesn’t add power
Run using a single actuator

Build faster robot by increasing power density
RAPID | APPROACH

Attach single motor to running mechanism
Mechanism translates power to running

VelociRoACH
Prior running robot
2 DoF, 43 W/kg, 2.7 m/s top speed
RAPID | APPROACH: MECHANSIM

To get running from open-loop power input:
Tune leg kinematics and stiffness to get stable motion

Establish mechanical stability
to get running by default

<table>
<thead>
<tr>
<th>Scaling Factor</th>
<th>Value $\alpha_X$</th>
<th>Cockroach Target</th>
<th>VelociRoACH Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length $\alpha_L$</td>
<td>3.3</td>
<td>3.4cm</td>
<td>11.2cm</td>
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<tr>
<td>Mass $\alpha_M$</td>
<td>36.1</td>
<td>0.83g</td>
<td>30g</td>
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<tr>
<td>Stiffness $\alpha_K$</td>
<td>--</td>
<td>--</td>
<td>40 N/m</td>
</tr>
<tr>
<td>Frequency $\alpha_\omega$</td>
<td>0.54</td>
<td>27 Hz</td>
<td>15 Hz</td>
</tr>
<tr>
<td>Velocity $\alpha_V$</td>
<td>1.2</td>
<td>1.5 m/s</td>
<td>2.72 m/s</td>
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<tr>
<td>Power $\alpha_P$</td>
<td>65.3</td>
<td>1.57 mW</td>
<td>103 mW</td>
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New running robot: X2-VelociRoACH
1 DoF, 120 W/kg

Single-motor design gets 3X power density
1: RAPID LOCOMOTION

Approach
Result
Top speed: 4.9 m/s
RAPID | RUNNING RESULTS (SLOWED 20X)

Footage captured at 600 fps

RAPID AND AGILE LOC MOTION
RAPID RUNNING RESULTS (SLOWED 19X)
1: RAPID LOCOMOTION

Approach
Result
Stride kinematics
Stride length increases at higher frequencies

Speed continues to increase with stride frequency
1: RAPID LOCOMOTION

Approach
Result
Stride kinematics
Force and power
Rapid | Predicted Power Draw

(a) A rendering of a Solidworks model of X2-VelociRoACH. The leg mechanisms have one degree of freedom, and are driven by a single virtual motor.

(b) Power requirements for driving the motor at constant rates, as a function of crank angle (see Fig. 4). The peak amplitude increases cubically with stride frequency.

Forces increase cubically with stride rate

Power increases cubically
Inertial forces drive power consumption

Power increases cubically, as predicted
Increased stride length caused by leg deformation
Top speed limited by self-destruction
1: RAPID LOCOMOTION

Approach
Result
Stride kinematics
Force and power
Energetics
Usually monotonically decreasing

Cubic power costs mean diminishing returns on efficiency
Demonstrated power-density first design strategy

- 1 Actuator driving specialized mechanism
- 3X increase in power density

X2-VelociRoACH is fastest running robot relative to size

Geometrically increasing power and damage motivate other methods for increasing speed
2: AGILE LOCOMOTION

INCREASING ROBOTIC AGILITY VIA POWER-DENSITY

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Energetics

2: AGILE LOCOMOTION
Motivation
FEMA USAR Training Site, Menlo Park
Need more mobility


AGILE | MOTIVATION

Ability to traverse rubble
• Not unstructured, but sparse

Small platform size
• More robust
• Won’t disturb site
• Cheaper

Imply leaping (saltatorial) locomotion
AGILE | ANIMAL SALTATORIAL LOCOMOTION

RAPID AND AGILE LOCOMOTION

Video by Nate Hunt
AGILE ROBOTIC SALTATORIAL LOCOMOTION

Running

[Park, Park, Kim 2015]

Jumping

[Kevas et al. 2008]

Controlled continuous locomotion
Low amplitude hops

Uncontrolled intermittent jumps
High-power jumps

OPEN AREA:
Controlled high-power jumps

RAPID AND AGILE LOCOMOTION
2: AGILE LOCOMOTION

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Motivation
Agility metric
To find animal that will inspire new robot:

Find metric that describes behavior
- Jump high
- Jump quickly

Metric:
Vertical Jumping Agility
AGILE | MODEL ANIMAL: GALAGO
AGILE | MODEL ANIMAL: GALAGO

Rapid and agile locomotion
AGILE | POWER DENSITY AND AGILITY

RAPID AND AGILE LOCMOTION
2: AGILE LOCOMOTION

INCREASING ROBOTIC AGILITY VIA POWER-DENSITY

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Motivation
Agility metric
Actuation strategies
Energetic goal: deliver as much jumping energy as possible

Approaches:

**Rigid**

**Parallel-elastic**

**Series-elastic**

Rigid maximum power determined by motor

Parallel-elastic can store/return energy. No maximum power limit (with latch)

Series-elastic can passively store/return energy. Increase max power by factor of 1.436
Saltatorial animals (and Salto)

Parallel-elastic

Series-elastic, Rigid

[Image 11x17 to 781x595]

AGILE | AGILITY METRIC

RAPID AND AGILE LOCMOTION

4/24/17

[Kovac et al, 2008]
2: AGILE LOCOMOTION

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SE+MA actuation
Energetic goal: deliver as much jumping energy as possible

Approaches:

- **Rigid**
- **Parallel-elastic**
- **Series-elastic**
- **Series-elastic + Mechanical advantage**

SE+MA strategy: add variable mechanical advantage to increase peak power
A variable mechanical advantage profile increases power modulation factor of series-elastic systems (Bullfrog study [Roberts, Marsh 2003])

Power modulation

\[ P_{\text{MAX}} > P_{\text{MOTOR}} \]

**Mechanical advantage:**

Output force

Input torque

Robotic system
Create hypothetical galago-sized robot, determine necessary power density

<table>
<thead>
<tr>
<th></th>
<th>Galago</th>
<th>Hypothetical rigid</th>
<th>Hypothetical series-elastic</th>
<th>Hypothetical parallel-elastic</th>
<th>Hypothetical SE+MA</th>
<th>EPFL Jumper</th>
<th>Salto</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (kg)</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.007</td>
<td>0.100</td>
</tr>
<tr>
<td>Leg Length (m)</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
<td>0.10</td>
<td>0.150</td>
</tr>
<tr>
<td>Maximum jump height (m)</td>
<td>1.74</td>
<td>1.74</td>
<td>1.74</td>
<td>1.74</td>
<td>1.74</td>
<td>1.38</td>
<td>1.008</td>
</tr>
<tr>
<td>Jump Frequency (Hz)</td>
<td>1.29</td>
<td>1.66</td>
<td>1.59</td>
<td>1.29</td>
<td>1.29</td>
<td>0.248</td>
<td>1.74</td>
</tr>
<tr>
<td>Vertical jumping agility (m/s)</td>
<td>2.24</td>
<td>2.89</td>
<td>2.78</td>
<td>2.24</td>
<td>2.24</td>
<td>0.34</td>
<td>1.75</td>
</tr>
<tr>
<td>Power density (W/kg)</td>
<td>92.7</td>
<td>343</td>
<td>325</td>
<td>21.9</td>
<td>90.0</td>
<td>50</td>
<td>137</td>
</tr>
</tbody>
</table>

SE+MA strategy reduces required power density by factor of 3.8

Parallel-elastic has lowest theoretical power density
2: AGILE LOCOMOTION

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- Leg mechanism
Design for power density: drive jump with single actuator
Leg mechanism has MA profile for power-modulation
and
Produces rotation-free jumps by default
AGILE | MECHANISM

REQUIREMENTS

1. The mechanism constrains a foot point to a vertical straight line, the line-of-action, in the frame of the robot.

2. Translation of the foot point, or stroke, is long relative to the size of the robot.

3. All pivots are located above the foot point at all times, with an input pivot near to the line-of-action.

4. Link lengths are compact.

5. The input link that attaches to the series elastic actuator rotates over a large range.

6. The leg possesses low mechanical advantage at the top of stroke.

7. Mechanical advantage defines a constant ground force for the remainder of stroke.

8. Moments exerted on the body of the robot by the mechanism are minimized.
Find all Stephenson linkages that trace a straight line
- Homotopy continuation method
- 4478 good solutions

Optimize start point from design space exploration to meet requirements


Prototype spins!
- Inaccurate mass models
- 6-bar hard to balance
AGILE | MECHANISM SYNTHESIS

6-bar hard to balance
⇒ Add two more links
⇒ Reoptimize


Final design satisfies all specifications
2: AGILE LOCOMOTION

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- Proof of concept
AGILE | ACTUATED PROOF OF CONCEPT

Rapid and agile locomotion
Mechanism

Binary tension links: *CF tie-rod*

Binary compression link, ternary links, body: *Pre-fab CF honeycomb panel*

Joints: *IGUS polymer bushings*

Connected by molded polyurethane

Panel and molds cut using *Othermill*

Spring

Torsional conic-sectioned latex

Actuator

COTS 3W DC gearhead
Increasing motor power → (Videos slowed 10X)
Power modulation

$P_{\text{MAX}} > P_{\text{MOTOR}}$

Power modulation factor: 3.63
AGILE | EFFECT OF POWER DENSITY

23.5 W/kg

58.8 W/kg

176 W/kg

RAPID AND AGILE LOCMOTION
2: AGILE LOCOMOTION

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Integrate high-power actuator, spring, inertial tail, and foot with jumping mechanism

[Libby et al 2012]
1.008 ± 0.007m vertical jumps,

Highest robotic vertical jumping agility of 1.75m/s
1.008 ± 0.007m vertical jumps,

Highest robotic vertical jumping agility of 1.75m/s

2.94x Greater peak power
AGILE | WALL JUMPS

4/24/17
Wall Jump Experiments: Slowed 10X

Novel wall-jump behavior
Wall jump height higher than max vertical from ground

Novel wall-jump behavior
AGILE | CONCLUSIONS

Agility metric able to compare animal and robot jumpers

Bio-inspired SE+MA strategy increases peak power

SE+MA can be done with singly-actuated mechanism

Salto has highest vertical jumping agility

High agility enabled the new wall-jump

Novel wall-jump behavior
3: SPATIAL AGILE LOCOMOTION

EXPANDING LOCOMOTOR CAPACITY WITHOUT COMPROMISING POWER DENSITY

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# 3: SPATIAL AGILE LOCOMOTION

EXPANDING LOCOMOTORY CAPACITY WITHOUT COMPROMISING POWER DENSITY

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Salto is only controllable in-plane

Unstable out of plane

To stabilize without compromising power density – Add thrusters (4.4 g)

<table>
<thead>
<tr>
<th></th>
<th>Salto</th>
<th>Salto-1P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (kg)</td>
<td>0.1000</td>
<td>0.0981</td>
</tr>
<tr>
<td>Active leg Length (m)</td>
<td>0.138</td>
<td>0.144</td>
</tr>
<tr>
<td>Maximum jump height (m)</td>
<td>1.007</td>
<td>1.252</td>
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<td>1.75</td>
<td>1.83</td>
</tr>
<tr>
<td>Max control torque (Nm):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pitch</td>
<td>0.029</td>
<td>0.034</td>
</tr>
<tr>
<td>Roll</td>
<td>0</td>
<td>0.0078</td>
</tr>
<tr>
<td>Yaw</td>
<td>0</td>
<td>0.0039</td>
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Novel wall-jump behavior
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*Max control torque (Nm):*
- Pitch: 0.029, 0.034
- Roll: 0, 0.0078
- Yaw: 0, 0.0039

Novel wall-jump behavior
3: SPATIAL AGILE LOCOMOTION

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3: SPATIAL-AGILE
- Attitude control
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AGILE | SPATIAL LOCOMOTION CONTROLLER

**Flight Phase**
- Orient for touchdown
- Retract leg

Positive spring deflection
- Leg reaches full extension

Negative spring deflection

**Equation**
\[
\theta = k_P y_{sat}(y_d - y, y_{max}) + k_V y (\dot{y}_d - \dot{y}) - k_V x (\dot{x}_d - \dot{x})
\]

Novel wall-jump behavior

[Image of robot and control diagram]

[Raibert 1984]
3: SPATIAL AGILE LOCOMOTION

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AGILE | SPATIAL IN-PLACE JUMPING

Rapid and agile locomotion

Novel wall-jump behavior
Novel wall-jump behavior
Novel wall-jump behavior
AGILE | SPATIAL IN-PLACE JUMPING

RAPID AND AGILE LOCMOTION
3: SPATIAL AGILE LOCOMOTION

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AGILE | SPATIAL FORWARDS- BACKWARDS JUMPING

Novel wall-jump behavior
AGILE | SPATIAL FORWARDS-BACKWARDS JUMPING

Novel wall-jump behavior
AGILE | SPATIAL FORWARDS-BACKWARD JUMPING

(a) Horizontal position of Salto-1P’s center of mass.
First jump establishes energy

63% average energy recovery by series elastic element
# 3: Spatial Agile Locomotion

Expanding locomotory capacity without compromising power density

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Novel wall-jump behavior
新型壁跳行为
Novel wall-jump behavior

8.93 m/s maximum Δv
AGILE | GRANULAR MEDIA

RAPID AND AGILE LOCMOTION
Power-density maintained using small actuators to get spatial control authority

Early work to simplify mechanism dynamics allowed use of simple Raibert controller

SE+MA mechanism able to passively recover 63% of jumping energy

Jumping power translates to wide range of $\Delta v$

Novel wall-jump behavior
OVERVIEW

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CONCLUSION | DESIGN FOR POWER-DENSITY

Tested power density first design with running robot and jumping robot.
Both were best in class.
QUESTIONS?

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RAPID AND AGILE LOCMOVEMENT
Kinematic Design of Salto’s Leg

Functions

- Transform motor torque into vertical jumping force
- Reduce required motor/gear train torque
- Minimize peak forces
- Jump without rotation
- Power modulate a series-elastic actuator

Requirements

1. Traces a straight line
2. Long stroke
3. CM near line-of-action
4. Compact dimensions
5. Input link rotates over large range
6. Low mech. adv. at top of stroke
7. Constant ground reaction force
8. Angular momentum balanced

Single structure
**Requirements**

1. Traces a straight line
2. Long stroke
3. CM near line-of-action
4. Compact dimensions
5. Input link rotates over large range
6. Low mech. adv. at top of stroke
7. Constant ground reaction force
8. Angular momentum balanced
Starting from Scratch

Requirements
1. Traces a straight line
2. Long stroke
3. CM near line-of-action
4. Compact dimensions
5. Input link rotates over large range
6. Low mech. adv. at top of stroke
7. Constant ground reaction force
8. Angular momentum balanced

We have no clue what this mechanism might look like.
A Prismatic Joint

A single spring-loaded prismatic joint

Requirements
✓ 1. Traces a straight line
✓ 2. Long stroke
✓ 3. CM near line-of-action
✓ 4. Compact dimensions
   5. Input link rotates over large range
   6. Low mech. adv. at top of stroke
   7. Constant ground reaction force
✓ 8. Angular momentum balanced

Sliders are heavy and not great with lateral loads.
Pick a four-bar

Use incredible visualization skills to design a four-bar:

Requirements
1. Traces a straight line
2. Long stroke
3. CM near line-of-action
4. Compact dimensions
5. Input link rotates over large range
6. Low mech. adv. at top of stroke
7. Constant ground reaction force
8. Angular momentum balanced
Straight-line Linkages

James Watt designed this linkage over 230 years ago:

Requirements
- 1. Traces a straight line
- 2. Long stroke
- 3. CM near line-of-action
- 4. Compact dimensions
- 5. Input link rotates over large range
- 6. Low mech. adv. at top of stroke
- 7. Constant ground reaction force
- 8. Angular momentum balanced

Other straight-line mechanisms:

Roberts linkage
Chebyshev linkage
Peaucellier linkage
An Atlas Of All Known Straight-Line Four-Bars

Four-bars w/ sliders: exact straight lines

Approx. straight line four-bars

- None of these have ground pivots in suitable locations

None satisfy our requirements


4/24/17
**Design Approach**

- **Required Behaviors**
- **Design Exploration**
- **Kinematic Tuning**
- **Final Design**

**Requirements**
1. Traces a straight line
2. Long stroke
3. CM near line-of-action
4. Compact dimensions
5. Input link rotates over large range
6. Low mech. adv. at top of stroke
7. Constant ground reaction force
8. Angular momentum balanced
Design Approach

Generate An Atlas Of Designs

Design Approach

1. Required Behaviors
2. Design Exploration
3. Kinematic Tuning
4. Final Design

Iterative Design Optimization

Design Approach

- Required Behaviors
- Design Exploration
- Kinematic Tuning
- Final Design

Key Design Phase

- Pre-existing designs not used for optimization starting points
- Does not depend on geometric intuition or “mechanical genius”
- Challenged by the curse of dimensionality
Design Exploration

Solving Massive Polynomial Systems

1. Motion requirements
   Example
   • It should go through these points
   • And be fast here
   • But slow here
   • And whatever else...

2. Polynomial system
   High order system with many solutions
   \[ f_1(x_1, \ldots, x_n) = 0 \]
   \[ f_2(x_1, \ldots, x_n) = 0 \]
   \[ f_3(x_1, \ldots, x_n) = 0 \]
   \[ f_4(x_1, \ldots, x_n) = 0 \]
   \[ f_5(x_1, \ldots, x_n) = 0 \]
   \[ f_6(x_1, \ldots, x_n) = 0 \]
   \[ \vdots \]
   \[ f_n(x_1, \ldots, x_n) = 0 \]

3. Polynomial solutions

Key process of design exploration:
Polynomial homotopy continuation